Key Points

- We have derived exact envelope equations governing the propagation and nonlinear interaction of short pulses

- We have applied the equations to second harmonic generation, optical parametric amplification, and synchronously-pumped optical parametric oscillators (OPOs).

- We have compared the solutions in several different approximations, including the slowly-varying envelope approximation (which is the least accurate).

- To study few-cycle effects, we have compared envelope and phase profiles for a range of different processes and parameter values.

- For the OPO, we calculate the absolute phase change between successive passes, an important diagnostic of few-cycle effects.

Approximations

Three different approximations have been used in this work. In order of increasing accuracy, these are:

SVEA

— Slowly-Varying Envelope Approximation

SEWA

— Slowly-Evolving Wave Approximation
— This is a first correction to the SVEA; see Brabec & Krausz, PRL 78 (1997) 3282, and also Porras, PRA 60 (1999) 5069.

GFEA

— Generalised Few-Cycle Envelope Approximation
— This is our new approximation for which we have derived the propagation equation; it includes all time-dependent corrections to the nonlinear polarization.

System Scaling

If the crystal length were fixed (as in a typical experiment), there would be significant differences between the results of simulations for different pulse durations, even for the same model (e.g. SVEA). This would make a systematic investigation of few-cycle effects impossible.

We therefore scale the crystal length and signal delay in direct proportion to the pulse length, and the pump energy in inverse proportion, i.e.

$$\frac{\text{Crystal Length}}{1000\mu\text{m}} = \frac{\text{Pulse width}}{48\text{fs}} = \frac{\text{Pump Pulse Energy}}{10\text{nJ}} = \frac{\text{Pump Delay}}{96\text{fs}}$$

This scaling means that a pulse of half the width has double the peak amplitude. The consequent increase in power counteracts the reduction in "few-cycle" effects resulting from the shorter crystal.

The scaling removes the gross effects of changing the pulse duration, so that those effects which remain are dependent on the choice of model. We expect to see increased few-cycle effects for shorter pulses when using the GFEA.

Conclusions

- The interaction dynamics are independent of the optical carrier phase, even for the shortest pulses.
- In the single-pass case, envelope and phase changes become increasingly significant for shorter pulses.
- In the multi-pass (OPO) case, the situation is more complex, and it is harder to discern systematic trends in the few-cycle effects ...
- ... however, a clear trend *can* be seen in the pass-to-pass changes in the envelope phase.