Few Cycle Pulses in an Optical Parametric Oscillator

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Talk; Abstract;

Motivation

♦ Few cycle pulses can be interesting in their own right.

♦ Few cycle pulses are being generated in the laboratory – *"Five-optical-cycle pulse generation..."* Beddard, etal, Opt.Lett.**25**, 1052 (2000).

★ Solving pulse propagation using Maxwell's equations is "easy" (e.g. FDTD), but envelope–based approaches are faster, and can be more intuitive.

★ Brabec & Krausz [PRL **78**, 3282 (1997)] derived an envelope SEWA eqn, extending the commonly used SVEA eqn into a few cycle regime.

★ Porras [PRA **60**, 5069 (1999)] derived a slightly different SEWA eqn to better treat transverse effects.

Summary

▶ Describe theory applicable to any nonlinearity.

▶ Apply the theory to various $\chi^{(2)}$ processes.

▶ Scaling rules: *vital* to ensure few-cycle effects are not obscured.

Theory (I)

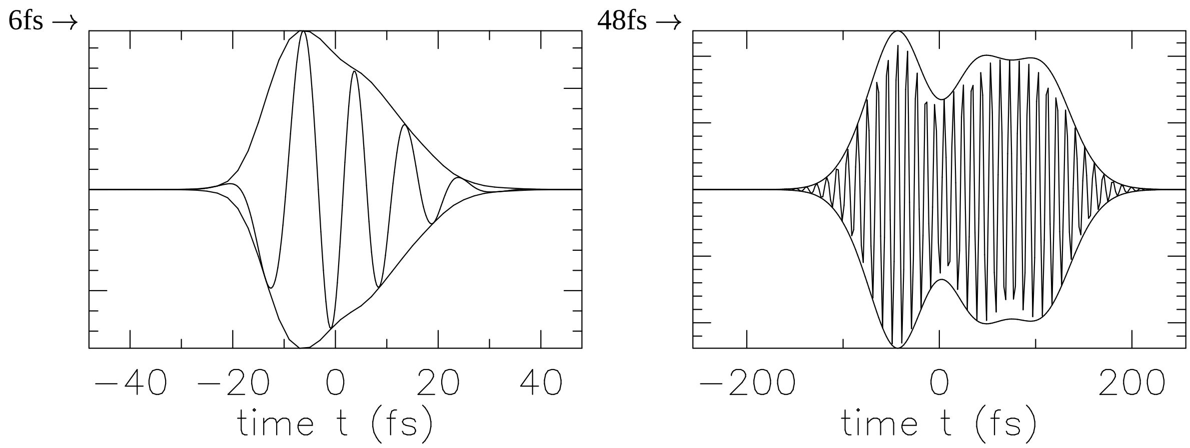
■ 3D wave equation, small transverse variation, plane polarized light, dispersion by expanding k about ω_0, with $k(\omega)^2 = \tilde{\varepsilon}(\omega)\omega^2/c^2$; z direction propagation, $\partial_\alpha \equiv \partial/\partial\alpha$:

$$\left(\partial_z^2 + \nabla_\perp^2\right) E(\vec{r},t) - \frac{\partial_t^2}{c^2}\int^t dt'\varepsilon(t-t')E(\vec{r},t') = \frac{4\pi}{c^2}\partial_t^2 P_{nl}(\vec{r},t)$$

■ Field ⇔ Envelope × Carrier

Split the descriptions of the field E (and polarization P_{nl}) into an envelope A (and B), and a carrier:

$$E(\vec{r},t) = A(\vec{r}_\perp,z,t)e^{\imath(\beta_0 z-\omega_0 t+\psi_0)} + \text{c.c.}$$
$$P_{nl}(\vec{r},t) = B(\vec{r}_\perp,z,t)e^{\imath(\beta_0 z-\omega_0 t+\psi_0)} + \text{c.c.},$$



► Pulse envelopes tend to be less useful in the few cycle case, because they are less likely to be smooth (w.r.t. ω_0).

► A given envelope represents many different field profiles, since there are many possible carrier phases.

Theory (II)

■ Derive an envelope propagation equation...

□ Split the description into A and A^* pieces, and solve one.

□ Co-moving variables $\tau = \omega_0 (t - \beta_1 z)$, $\xi = \beta_0 z$; define dimensionless $\sigma = \omega_0 \beta_1 / \beta_0 = (\omega_0/\beta_0)/(1/\beta_1) = v_f / v_g$; and use $n_0 = c\beta_0/\omega_0$.

□ Dispersion parameters $\gamma_n = \partial_\omega^n k(\omega)|_{\omega_\varepsilon} = \beta_n + \imath \alpha_n$.

$$(\beta_0/\omega_0)\hat{D}' = \left[\imath\alpha_1 (\imath\partial_\tau) + \sum_{n=2}^{\infty} \frac{\gamma_n \omega_0^{n-1}}{n!} (\imath\partial_\tau)^n \right].$$

■ Our exact propagation equation...

$$\partial_\xi A(\xi,\tau) = \left(-\frac{\alpha_0}{\beta_0} + \imath\hat{D}' \right) A(\xi,\tau) + \frac{\imath/\left(2\beta_0^2\right)}{(1+\imath\sigma\partial_\tau)} \nabla_\perp^2 A(\xi,\tau) + \frac{2\imath\pi}{n_0^2} \frac{(1+\imath\partial_\tau)^2}{(1+\imath\sigma\partial_\tau)} B(\xi,\tau;A) + \frac{T_{RHS}}{1+\imath\sigma\partial_\tau}.$$

▶ When $T_{RHS} = 0$, this is the generalised few-cycle approximation (GFEA) equation.

▶ Any backward propagating parts appear as a rapid modulation of the envelope; but would be approximated away when enforcing “smooth” envelopes.

Theory (III)

■ Approximations...

$$T_{RHS} = \left[-\frac{\imath}{2}\partial_\xi^2 + \frac{\imath}{2}\left(\frac{\alpha_0}{\beta_0} - \imath\hat{D}'\right)^2 \right] A(\xi,\tau) \approx 0.$$

Slow Evolution: $\partial_\xi^2 \ll 1$ **if** $\left|\partial_\xi \tilde{A}(\xi,\Omega)\right| \ll \left|\tilde{A}(\xi,\Omega)\right|$,

Weak Dispersion: $\partial_\tau \ll 1$ **if** $\left|\frac{\omega_0^m \gamma'_m \Omega^m}{\beta_0 m!}\tilde{A}(\xi,\Omega)\right| \ll \left|\tilde{A}(\xi,\Omega)\right|$,

Small Diffraction: $\nabla_\perp^2$ **if** $(1+\sigma\Omega)\,\beta_0^2 w_0^2 \gg 1$,

Weak Nonlinearity: B **if** $\frac{n_0^2}{2\pi}\frac{(1+\sigma\Omega)}{(1+\Omega)^2} \gg \frac{\left|\tilde{B}(\xi,\Omega;A)\right|}{\left|\tilde{A}(\xi,\Omega)\right|}$.

■ In order of increasing accuracy:

□ **SVEA:** *Slowly-Varying Envelope Approximation*

.
$$(1+\imath\partial_\tau)^2 / (1+\imath\sigma\partial_\tau)\, B \longrightarrow B \quad \text{SVEA}$$

□ **SEWA:** *Slowly-Evolving Wave Approximation*

— see Brabec & Krausz, and also Porras.

$$(1+\imath\partial_\tau)\left[1 + \frac{\imath(1-\sigma)\partial_\tau}{(1+\imath\sigma\partial_\tau)^2}\right] B \longrightarrow (1+\imath\partial_\tau)\, B \quad \text{SEWA}$$

$$\frac{(1+\imath\partial_\tau)^2}{(1+\imath\sigma\partial_\tau)} B \longrightarrow (1+\imath[2-\sigma]+\partial_\tau)\, B \quad \text{SEEA.}$$

□ **GFEA:** *Generalised Few-cycle Envelope Approx.*

— This is our new approximation; & includes all time-dependent corrections to the nonlinear polarization. Aka "Generalised Finite Envelope Approx" ⟶ FPEA – Finite Pulse Envelope Approximation

Notes (not for display)

I have ...

- ... derived exact envelope equations governing the propagation and nonlinear interaction of short pulses.
- ... done this without the usual SVEA derivation with its death by creeping approximation

This shows ...

- ... the extra "few cycle" effect is a derivative dependent phase adjustment to the nonlinearity, which will affect the phases of the evolving fields.

I now ...

- apply the equations to second harmonic generation (SHG), degenerate and non-degenerate optical parametric amplification (DPA, NPA), and synchronously-pumped optical parametric oscillators (OPOs).
- compare the solutions in several different approximations, including the (least accurate) slowly-varying envelope approximation (SVEA) and the best GFEA.
- study few-cycle effects, by comparing envelope and phase profiles for a range of different processes and parameter values.
- (in the OPO) calculate the absolute phase change between successive round trips, an important diagnostic of few-cycle effects.

Kerr Medium $\chi^{(3)}$

Brabec & Krausz tested their theory by testing its predictions against a full Maxwell Equations simulation, and got excellent agreement both with and without added dispersion.

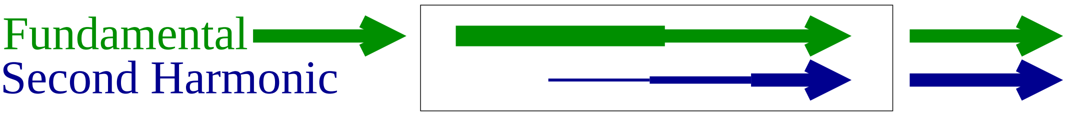
SVEA graph, SEWA graph, GFEA graph.

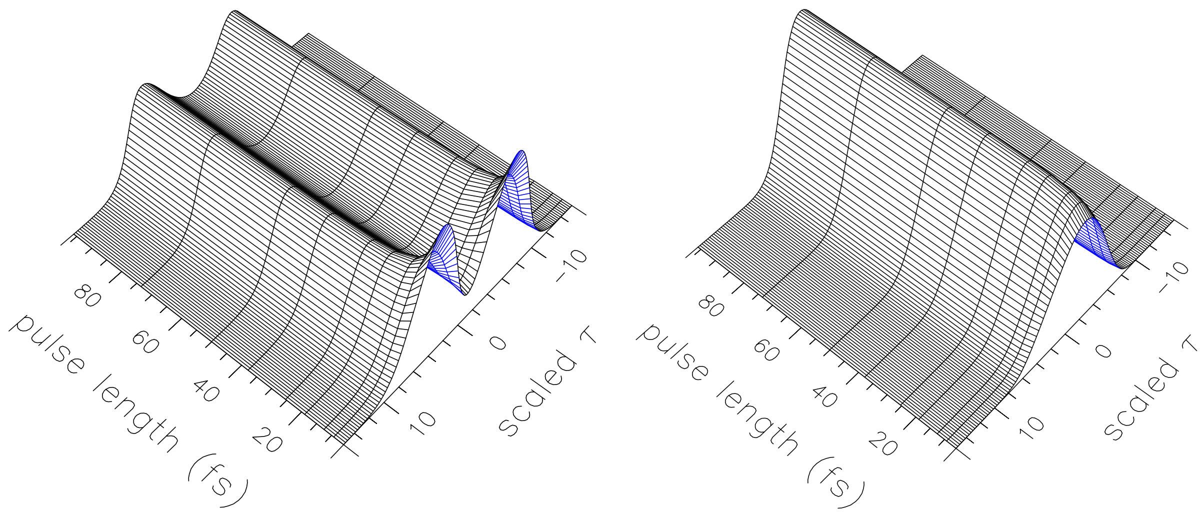
Ideal $\chi^{(2)}$ Notes (not for display)

- Few cycle effects uncluttered by dispersion – simulations on ideal $\chi^{(2)}$ crystal as a prelude to the full OPO simulations.
- Strong nonlinearity & conversion: depletion and repopulation.
- SHG (second harmonic):
  ▶ an input fundamental beam is converted into a second harmonic (i.e. frequency doubled) beam; ▶ eff. conversion eats a deep hole in the centre of the fundamental pulse; ▶ for short pulses, field phases get twisted and so the conversion is reduced.
- DPA and NPA (amplification):
  ▶ conversion from an input pump beam amplifies an input signal/idler beam; ▶ eff. conversion – signal centre gets depleted; ▶ for short pulses, field phases get twisted and so the conversion from (and the re-conversion to) the pump is reduced.
- DPD and NPD (de-amplification):
  ▶ conversion to an input pump beam de-amplifies an input signal/idler beam; ▶ eff. conversion – SVEA model nearly reaches the asymptotic state of perfect de-amplification; ▶ for short pulses, field phase twisting destroys the phase relationship needed for perfect de-amplification & *amplification* can occur; ▶ the residual SVEA signal also in the GFEA (use log scale);
- Scaling (I) ▶ matched τ and $|A|^2$ scales so that SVEA simulations no variation. ▶ The (unmarked) scales for the fundamental or pump $|A|^2$'s are larger than for the second harmonic/ signal/ idler modes.
- Scaling (II) the system parameters with pulse length to isolate few cycle effects – see later in the System Scaling section.

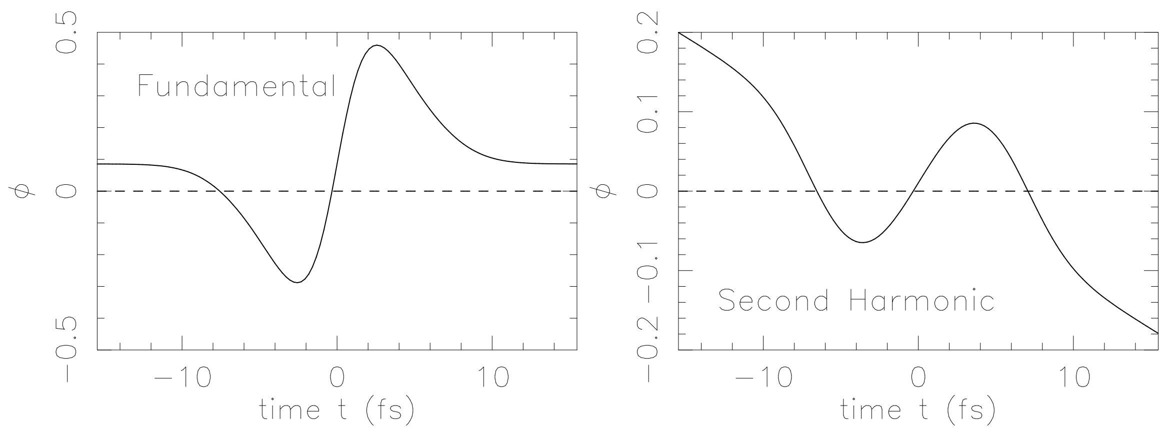
Second Harmonic Generation

Ideal dispersionless crystal – for example purposes only.





Scaled fundamental (left) and harmonic (right) pulse envelopes $|A|^2$. Pulse lengths are 6, 9, 12, 18, 24, 36, 48, 72, 96fs. SVEA results are like the 96fs result. See Notes.



Phase profiles for the pulse envelopes for an 18fs pulse duration, with a scaled τ. GFEA (——), SVEA (– – –).

► *What's odd about with these graphs? Much scaling!*

System Scaling

If the crystal length were fixed, as in a typical experiment, there would be significant differences between the results of simulations for different pulse durations, even for the same model (e.g. SVEA). This would make a systematic investigation of finite pulse length ("few-cycle") effects impossible.

We therefore scale the crystal length and signal delay in direct proportion to the pulse length, and the pump energy in inverse proportion, i.e.

$$\frac{\text{Crystal Length}}{1000\mu\text{m}} = \frac{\text{Pulse width}}{48\text{fs}} = \frac{10\text{nJ}}{\text{Pump Pulse Energy}} = \frac{\text{Pump Delay}}{96\text{fs}}$$

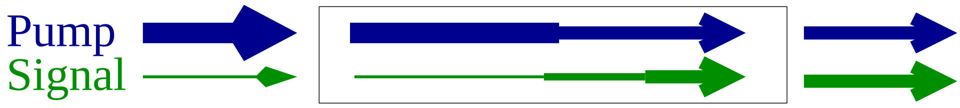
This scaling means that a pulse of half the width has double the peak amplitude. The consequent increase in power counteracts the reduction in "few-cycle" effects resulting from the shorter crystal.

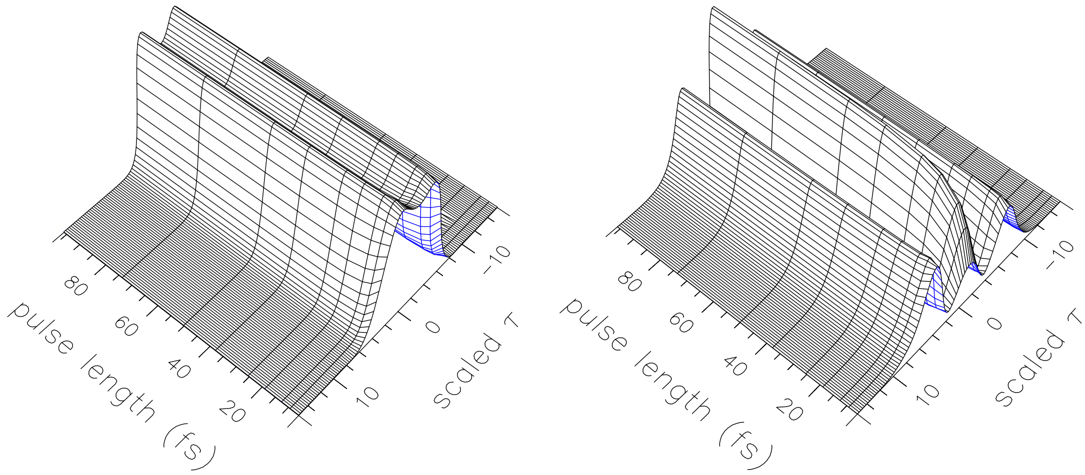
The scaling removes the gross effects of changing the pulse duration, so that those effects which remain are dependent on the choice of model. We expect to see increased few-cycle effects for shorter pulses when using the GFEA.

Note: We can get true pulse-length invariant propagation under the SVEA by also scaling the crystal dispersion: "super-scaling".

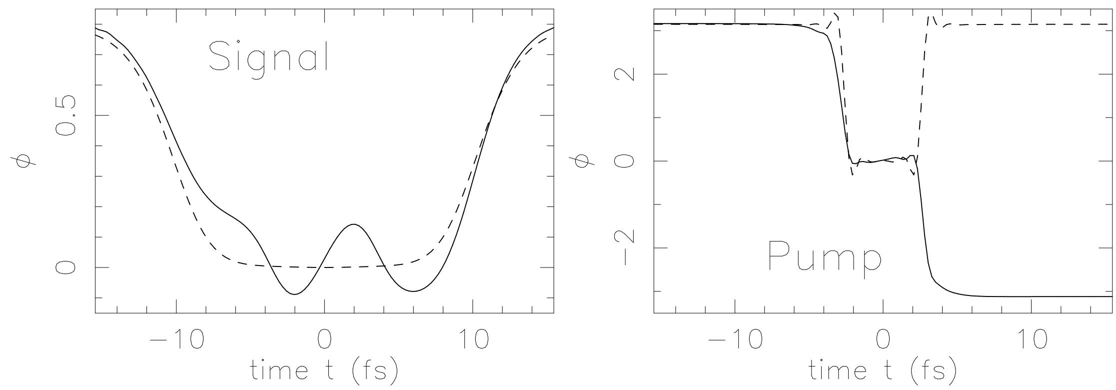
Degenerate Parametric Amplification

Ideal dispersionless crystal – for example purposes only.





Scaled pulse envelopes $|A|^2$ (left – signal, right – pump), for pulse lengths of 6, 9, 12, 18, 24, 36, 48, 72, 96fs. SVEA results are like the 96fs result. See Notes.

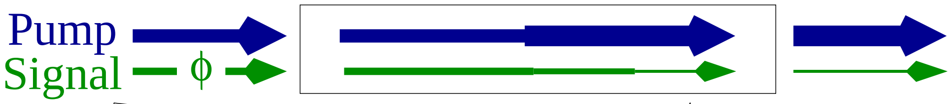


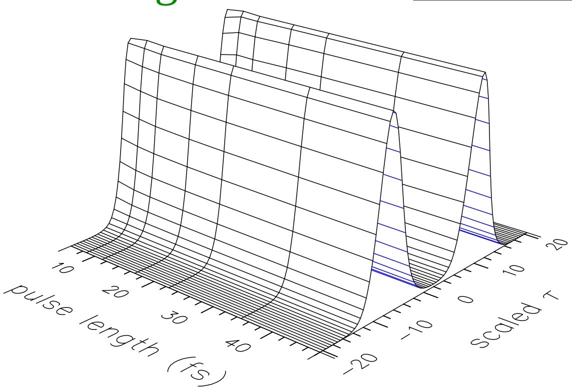
Phase profiles for envelopes for an 18fs pulse duration, with a scaled τ. GFEA (———), SVEA (– – –). The SVEA profiles are not flat because the system is phase sensitive w.r.t. the initial conditions.

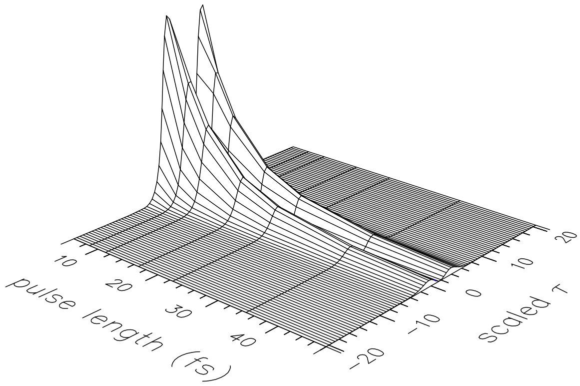
▶ *What's odd about with these graphs? Much scaling!*

Degen. Parametric De-amplification

Ideal dispersionless crystal – for example purposes only.

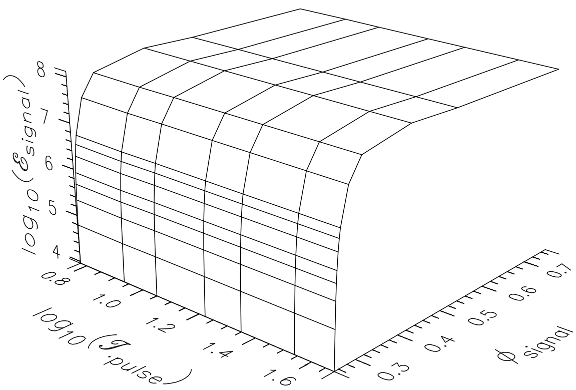


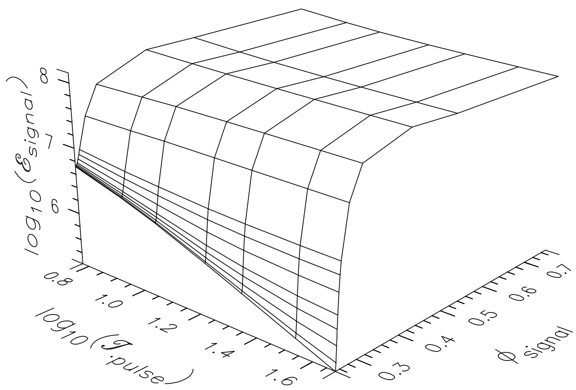




SVEA $\max|A|^2 \approx 400$; GFEA $\max|A|^2 \approx 400,000$.

- Output signal pulse $|A|^2$ for $\phi_s = \pi/4$ and $\phi_p = 0$:
- At 48fs, the peak GFEA intensity is a factor of ~ 32 larger than that of the SVEA model.



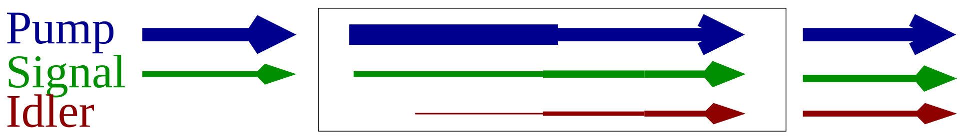


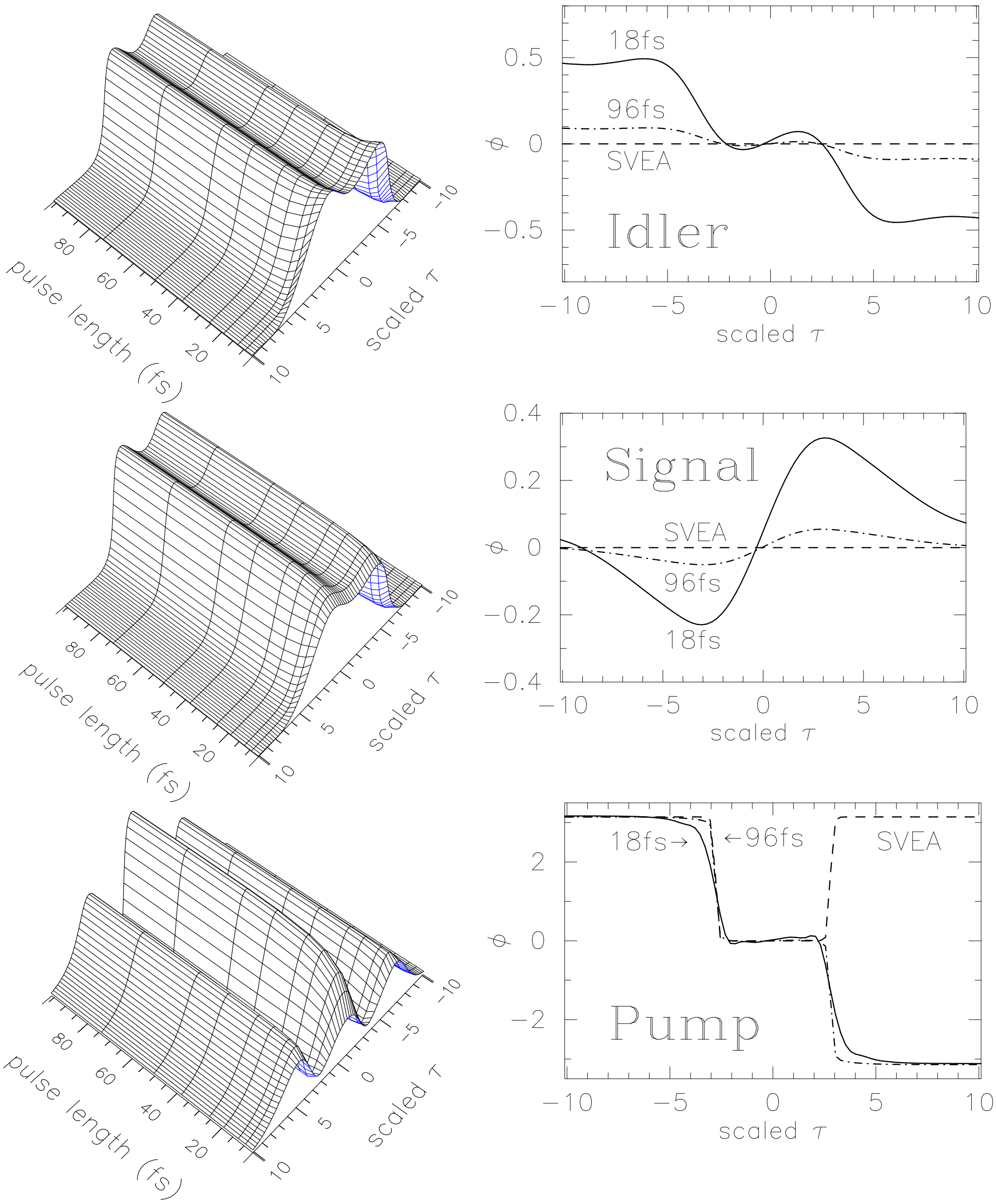
SVEA GFEA

Output signal pulse energies ($\mathcal{E}_{\text{signal}} = \int |A_{\text{signal}}|^2 d\tau$, arbitrary units) for a range of initial signal (envelope) phases ϕ_s and pulse lengths $\mathcal{T}_{\text{pulse}}$: $\mathcal{T}$ and $\mathcal{E}$ on log scales. $\phi_s \in \{0.250, 0.251, 0.252, 0.253, 0.254, 0.256, 0.258, 0.260, 0275, 0300, ...\}$

Parametric Amplification

Ideal dispersionless crystal – for example purposes only.

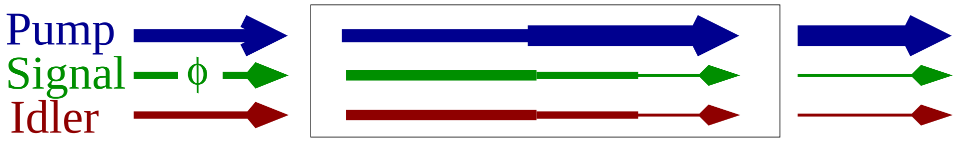


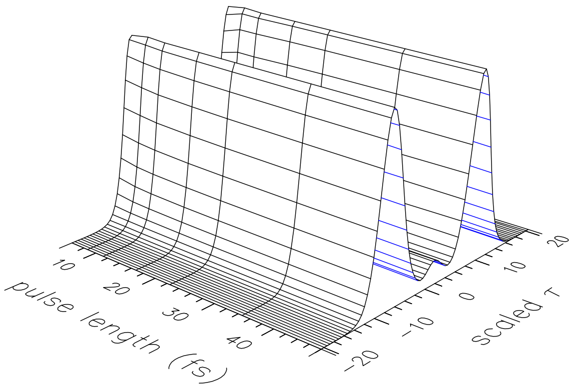


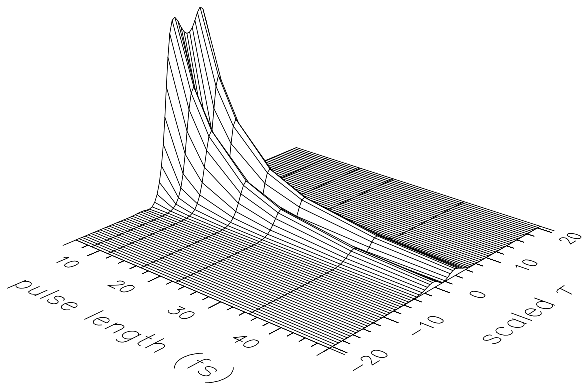
Scaled pulse envelopes ($|A|^2$, left) and phase profiles (right), for pulse lengths of 6 – 96fs. The SVEA envelopes are very similar to the 96fs GFEA envelopes. See Notes.

Parametric De-amplification

Ideal dispersionless crystal – for example purposes only.

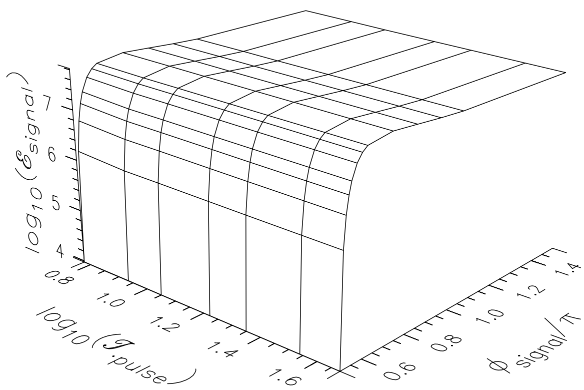




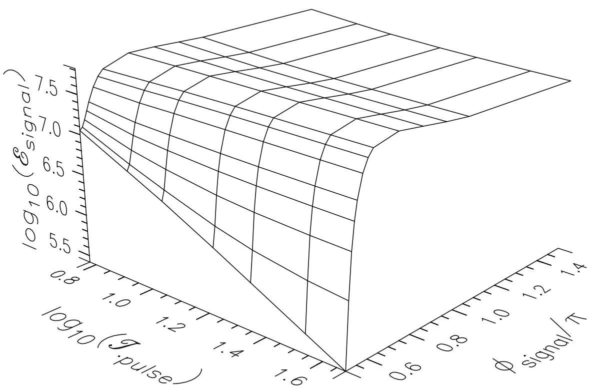


SVEA $\max|A|^2 \approx 400$; GFEA $\max|A|^2 \approx 600,000$.

- Output signal intensities for $\phi_s = \pi/2$ and $\phi_p = \phi_i = 0$:
- At 48fs, the peak GFEA intensity is a factor of ~ 32 larger than that of the SVEA model.



SVEA



GFEA

Output signal pulse energies ($\mathcal{E}_{\text{signal}} = \int |A_{\text{signal}}|^2 d\tau$, arbitrary units) for a range of initial signal (envelope) phases ϕ_s and pulse lengths $\mathcal{T}_{\text{pulse}}$: $\mathcal{T}$ and $\mathcal{E}$ on log scales.

$\phi_s \in \{0.500, 0.510, 0.520, 0.530, 0.540, 0.560, 0.580, 0.600, \ldots\}$

OPO Notes (not for display)

■ OPO diagram: injected pump interacts with signal, enerating an idler. The pump and idler exit the cavity, but the signal is reflected for the next round trip.

■ OPO: ▷▷ 0.8400μm ↔ 357THz; ▷▷ 1.1650μm ↔ 258THz; ▷▷ 3.0110μm ↔ 99THz.

■ $LiNO_3$: Sellmeier eqn., Jundt, Opt. Lett. **22**, 1553 (1997) , hence: ▶ different group velocities: ▷▷ pump is slower, so pump input time is ahead of signal; ▷▷ pump catches signal up about halfway down the crystal; ▷▷ pump-signal overlap generates idler

■ Use (to start with) System Scaling rules to clarify "few cycle" effects – hence "scaled OPO".

■ Envelopes: ▶ differences in pulse shapes from 6fs ⟶ 36fs, but *no clear trend*; ▶ differences between SVEA & GFEA, but *no clear trend*; ▶ 48fs unstable.

■ No Clear Trend? Effects of few cycle terms are too small or too subtle to show through – the OPO constantly recycles the signal pulse, so it is a very complex system. Dispersion not scaled, so SVEA not pulse-length invariant.

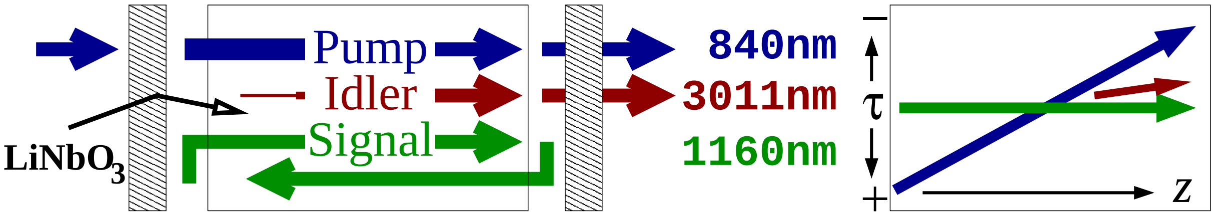
□ Super-Scaled OPO: here I also scale the dispersion, so the SVEA simulations are pulse-length scale invariant. The GFEA results undergo a transition from one pulse profile between 36fs → 48fs, and then converge to the SVEA.

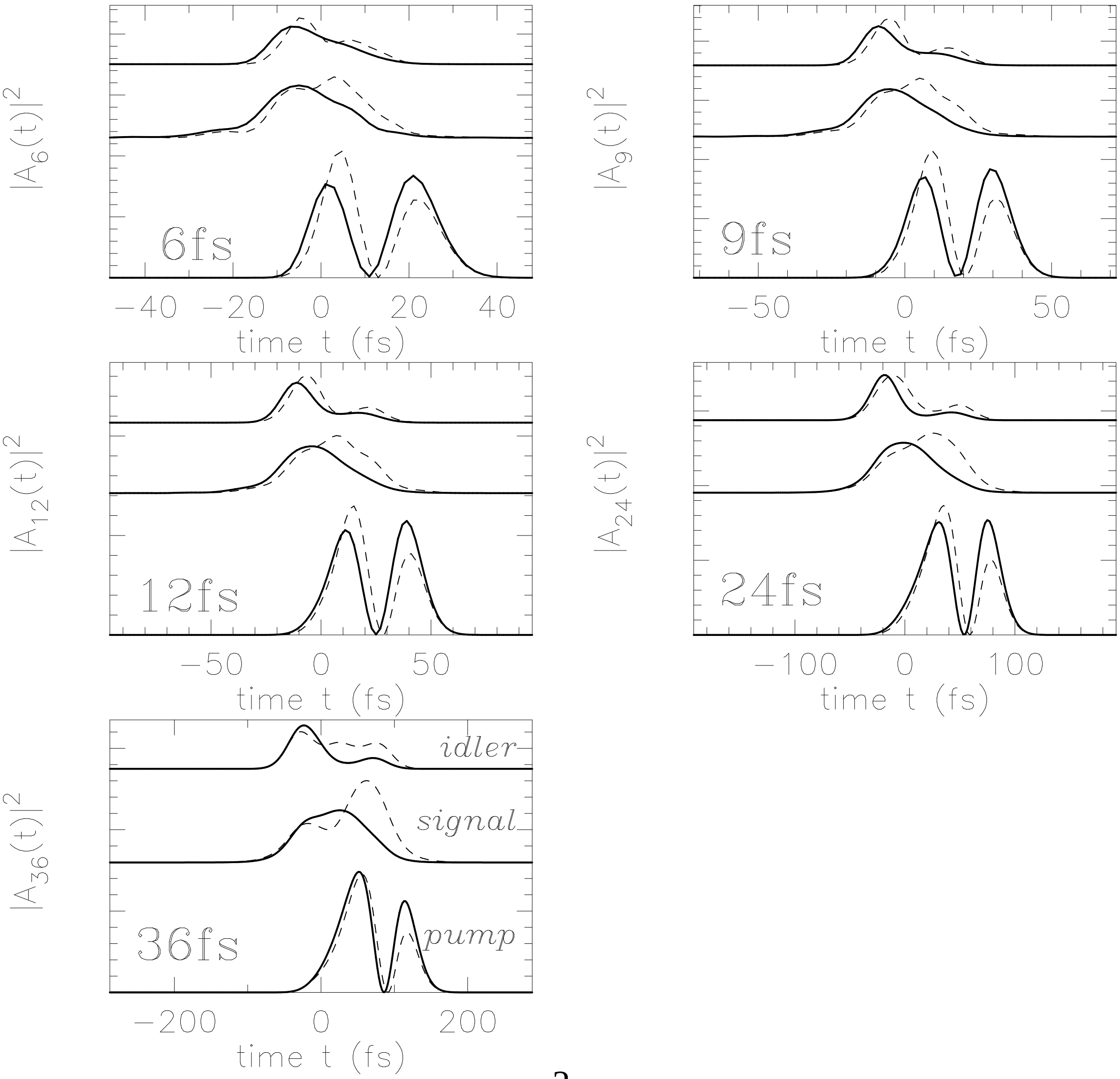
□ Cyles: at 6fs, the idler is clearly in the few cycle regime – also at 9fs, 12fs.

□ Spectra: get broader in inverse proportion to the pump pulse width – as expected – 6fs, 12fs, 48fs. At 6fs the sspectra nearly overlap – breakdown of independent-field assumptions.

■ Phase drift: ▶ Difference in size and character of phase drift between SVEA & GFEA ▶ Lower graphs – fixed length crystal – show the difference in phase drift ($\Delta\phi_{pp}$) effect is most important for better phase matched systems.

Scaled Optical Parametric Oscillator



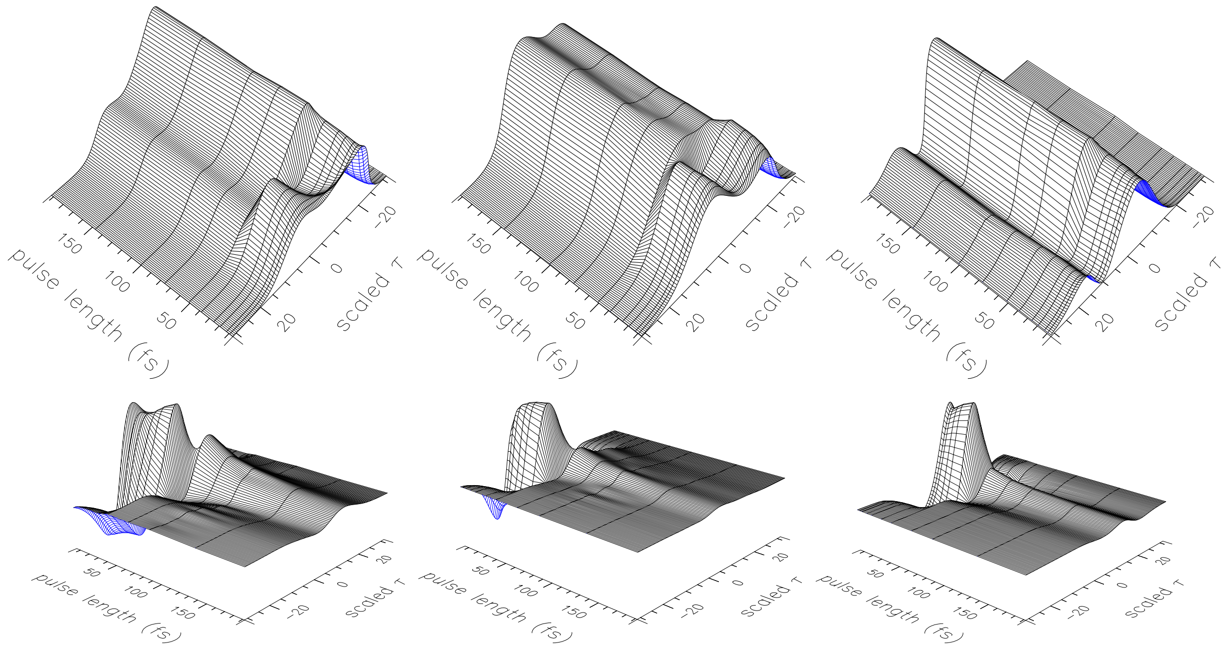


Scaled pulse envelopes $|A|^2$, for 6fs – 36fs. See Notes.

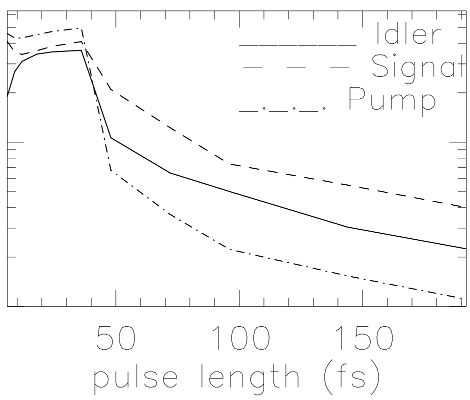
- In each frame, lower to upper: pump, signal, idler.
- GFEA (——) and SVEA (– – –).
- Note variation in SVEA and GFEA profiles.

Super-Scaled OPO

With scaled crystal dispersion – for example purposes only.

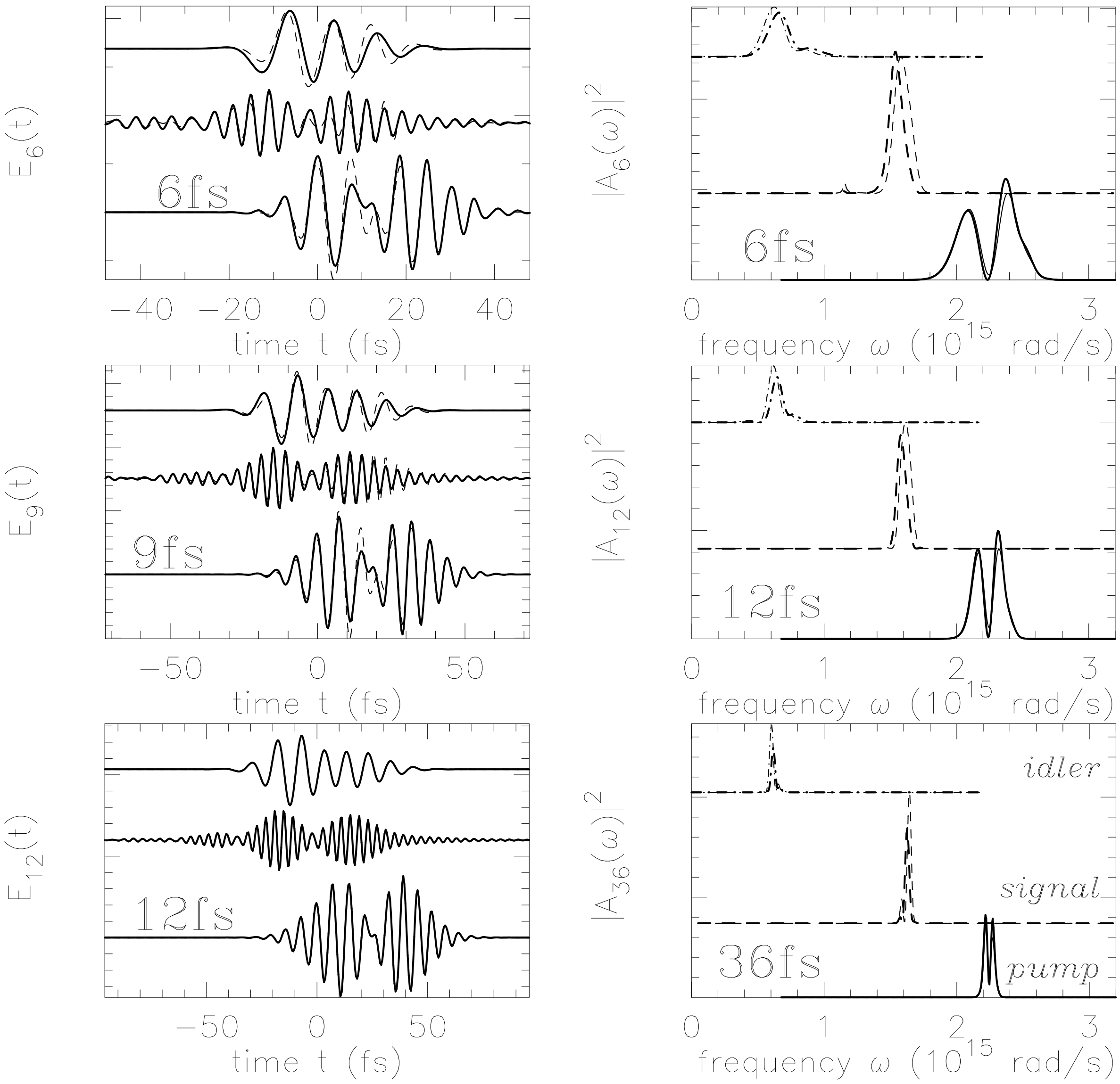


Scaled GFEA pulse envelopes for 6fs – 192fs and super-scaled parameters. Note the transition at ∼40fs. *All* SVEA envelopes are very similar to the 192fs GFEA. Left–Right: idler, signal, pump; Top: GFEA envelopes; Bottom: SVEA – GFEA difference.



Convergence of GFEA and SVEA simulations for longer pulse lengths: maximum difference over the middle quarter of the scaled τ range, on a $\log_{10}$ scale. See Notes.

Scaled OPO: cycles and spectra



LEFT: $E(t)$: top to bottom, 6, 9, 12fs; chosen carriers.

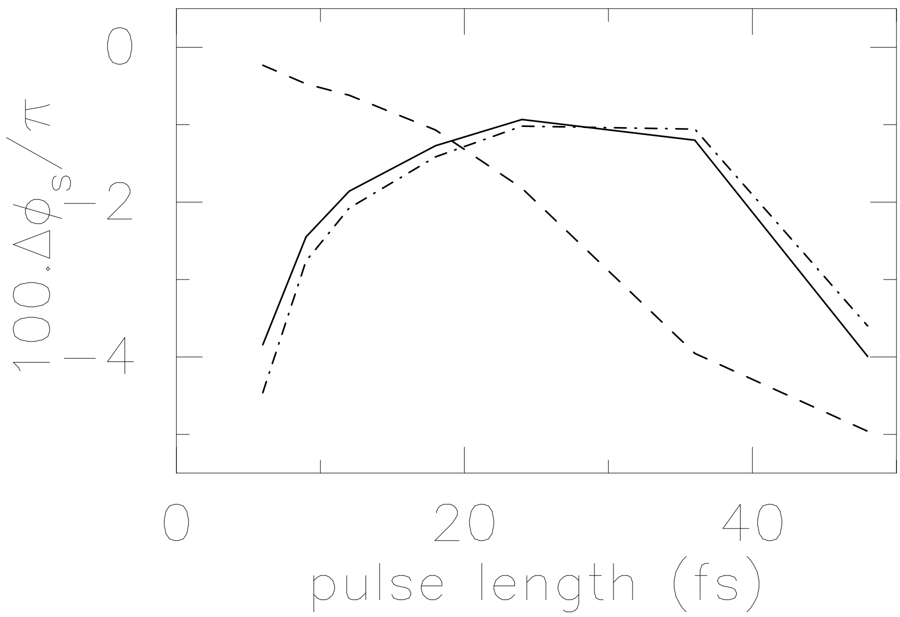
RIGHT: $\tilde{E}(\omega)$ top to bottom, field spectra; 6, 12, 36fs.

- In each frame, lower to upper: pump, signal, idler.
- GFEA (—) and SVEA (– – –).

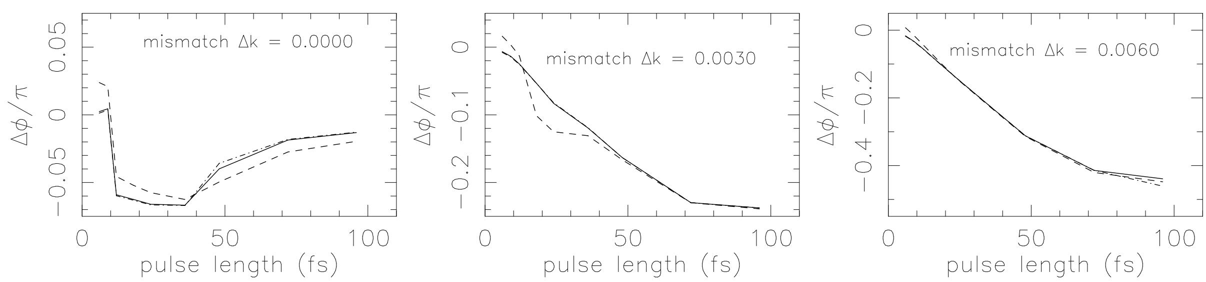
See Notes.

Scaled OPO: phase drifts

At steady state, after each pass of the oscillator, the $|A|^2$ for each field are the same. However, the phase of the signal and idler envelopes have advanced by a fixed amount.



Phase drifts for a range of pulse lengths and perfect phase matching, GFEA (——), SEWA (–·–·–·), and SVEA (– – –). The differences are taken between the phase at the peak of the modulus-squared of the signal envelopes between passes through the crystal.



Conclusions

▶ **Theory:** interaction dynamics are independent of the optical carrier phase, even for the shortest pulses.

▶ **Theory:** few-cycle effects add an extra phase twist to the nonlinearity.

▶ **Scaling:** *vital* for seeing few-cycle trends – which would otherwise be swamped by other phenomena.

★ **Ideal $\chi^{(2)}$:** envelope and phase changes due to the finite pulse lengths ("few-cycle effects") become increasingly significant for shorter pulses.

★ **Ideal $\chi^{(2)}$:** Remarkable differences can be seen for "de-amplification" of signal pulses in when finite pulse lengths ("few-cycle effects") are included.

⧫ **OPO:** (multi-pass) this situation is more complex, and it is hard to discern systematic trends in $|A|^2$ due to few-cycle effects ...

⧫ **OPO:** ... however, a clear trend *can* be seen in the pass-to-pass changes in the envelope phase – these phase drifts are more significant the better phase-matched the system is.

▶ **Next:** Cavity synchronisation ... Transverse effects ... Look for finite pulse length effects in a non-few-cycle regime in realistic systems.