# Testing quantum mechanics using third order correlations.

Paul Kinsler
*Department of Physics∗, University of Sheffield, Sheffield S3 7RH, United Kingdom.*

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Semiclassical theories similar to stochastic electrodynamics are widely used in optics. The distinguishing feature of such theories is that random statistical fluctuations are used to model the quantum uncertainty. These random statistical fluctuations act like hidden variables. They can successfully predict some quantum mechanical phenomena for example, the squeezing of the quantum noise in the parametric oscillator. However, such theories are not equivalent to quantum mechanics. Complex number representations can be used to to exactly model the quantum uncertainty, but care has to be taken that approximations do not reduce the description to a hidden variable one. The quantum and hidden variable theories make very different predictions for higher order correlations in non linear systems.

Third order correlations can provide a test that shows a clear distinction between quantum and hidden variable theories in a way analogous to that provided by the 'all or nothing' GHZ test of local hidden variable theories.

[*] I worked at this institution when presenting this work at NQEC in Southampton, U.K., in 1995; but the content was largely derived from my PhD thesis done at the University of Queensland in Australia with Prof. Peter Drummond.

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## I. A ONE PAGE SUMMARY

Third order correlations can provide a test that shows a clear distinction between quantum and hidden variable theories in a way analogous to that provided by the 'all or nothing' GHZ test of local hidden variable theories. In Bells tests, the EPR argument gives a lower bound for a hidden variable theory that is larger than the lower bound given by quantum mechanics.

The system investigated here is the degenerate parametric oscillator, and the focus is on high order moments of the fields.

The stochastic electrodynamics (SED) theory is constructed by taking classical equations for the fields and adding a noise term to each. This noise plays the same role as the quantum uncertainty does in quantum mechanics, but is equivalent to a hidden variable theory. The predicted behaviour of the third order moment given by the SED theory is a linear increase for short times.

A quantum calculation for the third order moment predicts no increase for short times, clearly showing the difference between the two theories. The difference between the hidden variable (SED) and quantum result is evident for all initial photon numbers.

These results suggest new tests measuring third order correlations that could distinguish between quantum and hidden variable theories in a dramatic way.

## II. INTRODUCTION

Local hidden variable theories give predictions incompatible with quantum mechanics in Bells example, the EinsteinPodolskyRosen [EinsteinPR1935] argument gives a lower bound larger than the lower bound given by quantum mechanics

[Mermin1981].

Stochastic electrodynamics (SED) theory is equivalent to the phenomenological approach of taking a classical model and adding some noise intended to represent the quantum mechanical vacuum fluctuations [Marshall1963, Boyer1980]. This "stochastic electrodynamics" is a hidden variable theory, because it represents the quantum uncertainty as a classical probability distribution over electric field amplitudes.

Recently, new tests of local hidden variable theories have been proposed [GreenbergerHZ1989]. The GHZ proposal is of a new type, and has an "all or nothing" outcome.

The prediction for the difference between local hidden variable theories and quantum mechanics is no longer statistical. In principle it can be tested in a single attempt by using differences between three particle correlation functions for local hidden variable and quantum theories. These new tests suggest that analogous higher order tests on quadrature measurements could produce similar tests of quantum mechanics. The original EPR paradox has been demonstrated experimentally using quadrature correlations in the parametric oscillator [OuPK1992].

Here I present calculations [Kinsler1994, 1995] showing distinct differences between a particular third order moment for the SED and quantum theories.

Complex number distributions can be used to represent quantum mechanical systems. Different representations describe the same quantum uncertainty in different ways the positiveP representation uses a basis of quantum mechanical coherent states, and so the quantum uncertainty is implicit in the description. In contrast, the more widely known Wigner representation is a distribution over classical field amplitudes. This means the quantum uncertainty is represented by the distribution itself and so approximations can reduce it to a hidden variable theory equivalent to SED. It is a hidden variable because it involves a probability distribution over field amplitudes. This type of theory is unable to produce anything like the correct moments in the system considered here.

The third order correlation (or moment) results in this poster suggest that it should be possible to test quantum mechanics using quadrature measurements on the macroscopic photon states produced using non linear optical materials.

## III. THE MODEL

The system investigated here is the degenerate parametric oscillator, and the focus is on high order moments of the fields.

Cantrell, Lax and Smith [1973] have calculated the third and higher order intensity correlations of laser light near threshold. These were measured experimentally by Corti and DeGiorgio [CortiD1974, 1976].

Here I consider third order moments of the field rather than intensity moments, which can be done with the use of a local oscillator. Third order moments are correlations with a zero time difference.

The system considered is a degenerate parametric oscillator [Bloembergen1965, DrummondMW1981], described as an idealised interferometer that is resonant at two frequencies, w1 and wp. The electromagnetic field mode containing the higher frequency photons is usually called the pump mode ($a_p$), and the lower frequency mode the signal mode ($a_1$). Down conversion of the pump photons to the sub harmonic mode occurs due to a $\chi^{(2)}$ non linearity present inside the cavity.

The initial conditions I choose for this model are a coherent state in the pump mode with amplitude $\varepsilon$, and a vacuum state in the signal mode. The quantum Hamiltonian in the interaction picture is

$$H = ik\left[a_1^2 a_p\dagger - a_1\dagger^2 a_p\right] \tag{3.1}$$

The stochastic electrodynamics field variables that correspond to the operators $a_1, a_p$ will be denoted $b_1, b_p$.

## IV. THE UNCERTAINTY

The stochastic electrodynamics (SED) theory is constructed by taking classical equations and adding random fluctuations. These fluctuations play the same role as the quantum uncertainty does in quantum mechanics, but make it the same as a hidden variable theory.

In this approach, SED is presented as a phenomenological extension to classical optics. The state of the system is represented by an ensemble of equivalent subsystems where the uncertainty is built into the initial conditions of the ensemble.

We can compare a FokkerPlanck equation for the SED theory to the one for the quantum Wigner function [Wigner1932]. The Wigner function represents the quantum states as a quasidistribution, and quantum effects can be described by this since it can take on negative values.

The only difference between the Wigner and the stochastic electrodynamics FokkerPlanck equations is that the Wigner equation has extra third order derivative terms. The form of these terms suggests that comparing the third order moment for the quantum system

$$M = (a_1 - < a_1 >)^2 \, (a_p\dagger - < a_p\dagger >) \,, \tag{4.1}$$

to that for the corresponding SED system

$$M_S = (b_1 - < b_1 >)^2 \, (b_p * - < b_p * >) \,, \tag{4.2}$$

would most clearly highlight the differences between the two theories.

## V. HIDDEN VARIABLES

# What is the predicted behaviour of the third order moment given by the hidden variable, stochastic electrodynamics theory?

I start with the classical equations of motion for the field amplitudes. The time is scaled by the coupling constant $t = kt$. The equations of motion for the stochastic electrodynamics (SED) description of the system are just the classical equations, and are

$$db_1/dt = b_1 * b_p, \tag{5.1}$$

$$db_p/dt = -b_1^2/2. \tag{5.2}$$

The difference in these SED equations is that they need to be averaged over an ensemble of initial conditions. These are distributed in a gaussian way around the average. I leave the details of the moment calculation to the preprint [Kinsler1995].

For the chosen initial conditions, the first order increment in the moment $M_S$ is

$$d < MS >= dt/4 + O(dt2). \tag{5.3}$$

This hidden variable, SED theory predicts an initial linear increase in the value of the moment. Now I will compare this result to the quantum mechanical predictions.

## VI. QUANTUM MECHANICS

# A quantum calculation is done in order to compare the quantum moment with the hidden variable, stochastic electrodynamics result.

Here I present the results of quantum mechanical calculations for the time evolution of the third order moment. The time is scaled by the coupling constant $t = kt$. I proceed by substituting the time evolved state into the operator expression for the third order moment, and hence obtain an equation of motion for the expectation value. This is then evaluated in the short time limit.

Leaving the details to the preprint [Kinsler1995], the change in the moment $M$ to first order is

$$dM = O(dt^2) \tag{6.1}$$

Another way of doing a quantum calculation of the evolution of the third order moment M is using the positiveP representation [DrummondG1980]. This describes each field mode amplitude by two complex numbers. The two complex numbers result from the way the representation expands the density matrix for the fields using an off diagonal basis of coherent states. It is always possible to find a positiveP function for the quantum state that is also a probability distribution.

The variables $a_1, a_1\dagger$ describe the scaled field amplitude of the signal mode; and $a_p, a_p\dagger$ describe the pump mode. The stochastic differential equations for them are

$$da_1 = a_1\dagger a_p dt + a_p dW_1, \tag{6.2}$$

$$da_1 = a_1 a_p\dagger dt + a_p\dagger dW_2, \tag{6.3}$$

$$da_p = -a_1/^2/2dt, \tag{6.4}$$

$$da_p\dagger = -a1\dagger^2/2dt, \tag{6.5}$$

where $< dW_i dW_j >= d_{ij} dt$. These $dW_i$ are gaussian white noise increments. The initial state is a vacuum in the signal and a coherent state of amplitude e in the pump. In the positiveP representation, the initial conditions of both modes can be represented by delta functions.

The quantum moment written in the positive-P notation is

$$MP = (a_1 - < a_1 >)^2 (a_p\dagger - < a_p\dagger >), \tag{6.6}$$

I do the calculation in a similar way to the SED calculation, but again leave the details to the preprint [Kinsler1995].

To first order the ensemble average of the change in the moment is

$$< dM_P >= O(dt^2) \quad (6.7)$$

If we iterate this positive-P moment to higher order, we find that it is zero to the next order as well. This predicted moment agrees with the operator prediction, but not with that of the SED calculation.

This is because the quantum uncertainty is built into the coherent state basis used to generate the positiveP representation. This contrasts with the SED theory, which treats the uncertainty like a random hidden variable.

## VII. DISCUSSION

The results have clearly shown the difference between the stochastic electrodynamics (SED) theory and the quantum mechanical theories.

SED predicts that the third order moment considered changes with time as dt, whereas for the quantum calculations the change is zero. This might be regarded as a somewhat surprising result, given the "standard view" that quantum and classical results should be the same for very short times [BermanYZ1991]. However, examination of the SED calculations reveals that the initial increase in the moment is due to the effect of the vacuum noise term – in particular, to the fourth moment of the noise.

A noiseless classical theory would give a result the same as the quantum for short times, but of course the long time behaviour would now be hopelessly wrong.

The evolution with time of the real part of the third order moments is shown on figure 1. It shows the numerical results for a range of initial coherent state amplitudes. From these graphs, which compare the positiveP simulation results $< M_P >$ to the corresponding SED results $< M_S >$, it would seem that it does not agree with the quantum theory at all for low photon numbers, but does agree for larger photon numbers. The initial slope of the $< M_S >$ result is $1/4$, as predicted by the theory.

Highlighting the initial behaviour (figure 2) shows that the difference between the hidden variable (SED) and quantum results is evident for all initial photon numbers.

Figures 2(ad) use the same data as in figures 1(ad).

## VIII. CONCLUSION

Here I have clearly demonstrated the limits of a hidden variable stochastic electrodynamics approach to non linear quantum systems.

This was achieved by obtaining exact quantum results and comparing them to the corresponding stochastic electrodynamics SED calculations. The results showed a clear difference between the two theories, the hidden variable one disagreeing markedly with the quantum evolution. This difference occurred for all photon numbers – even in the usual large photon number "semiclassical" limit SED did not make predictions compatible with those of quantum mechanics.

The quantum vacuum is often referred to as being a state of zero amplitude, with some fluctuations or "noise" added. These results clearly show that this is not a 'classical' type of noise.

Of course, such conclusions can also be drawn from studies of Bells inequality violations, where "local realism" is shown to be incompatible with quantum mechanics. "Local realism" gives rise to hidden variable theories where the quantum uncertainty is assumed to be due to some unmeasurable part of the system, just as in SED.

The difference in the moments suggests the possibility of new tests that measure third order correlations, which could distinguish between quantum and hidden variable theories in a dramatic way.

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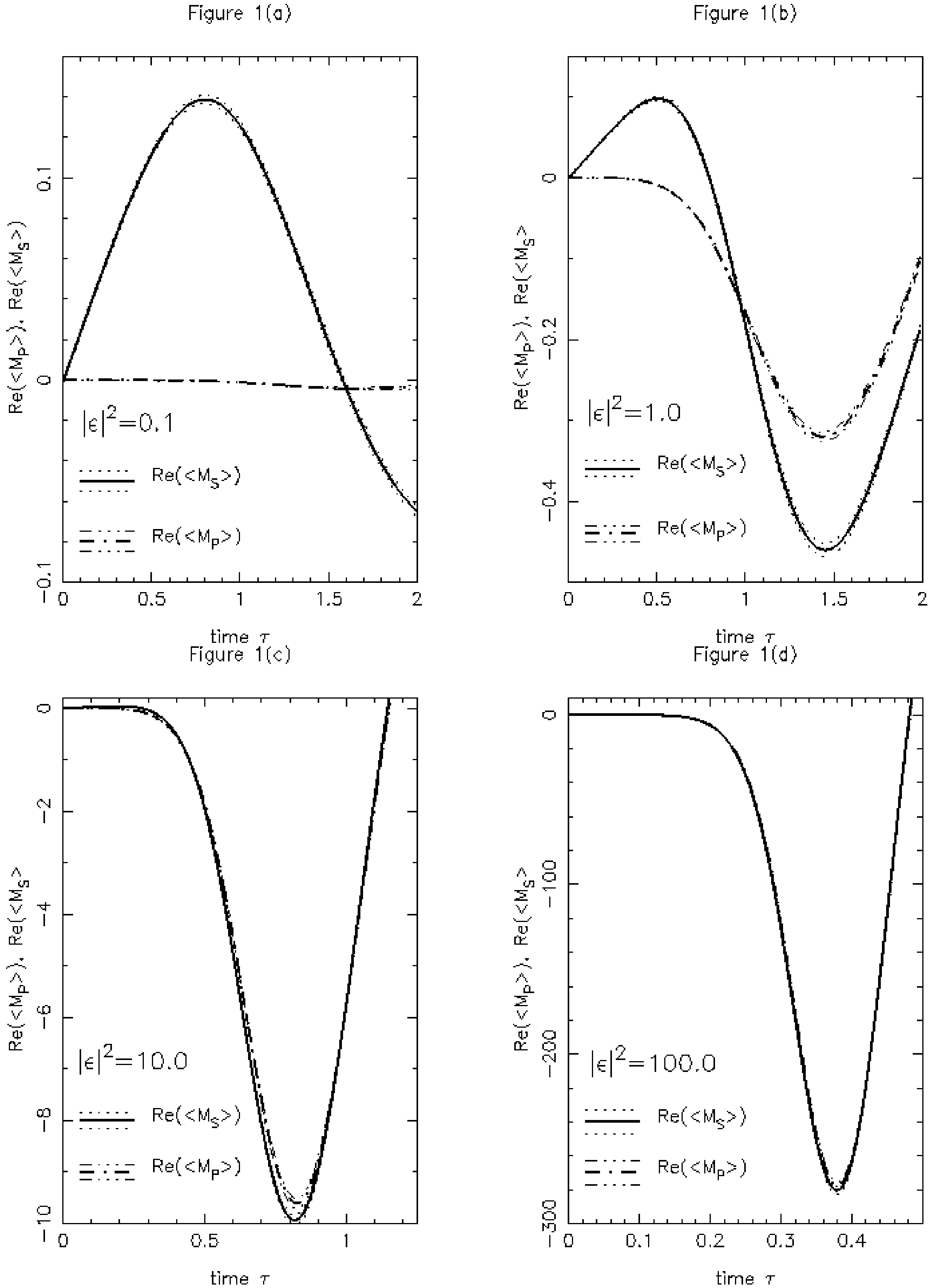


FIG. 1: **Figure 1:** Graphs showing the evolution of the third order moments $M_S, M_P$ as a function of time for a range of initial conditions. The statistical errors due to the finite ensembles are indicated by the broken curves above and below the average values.

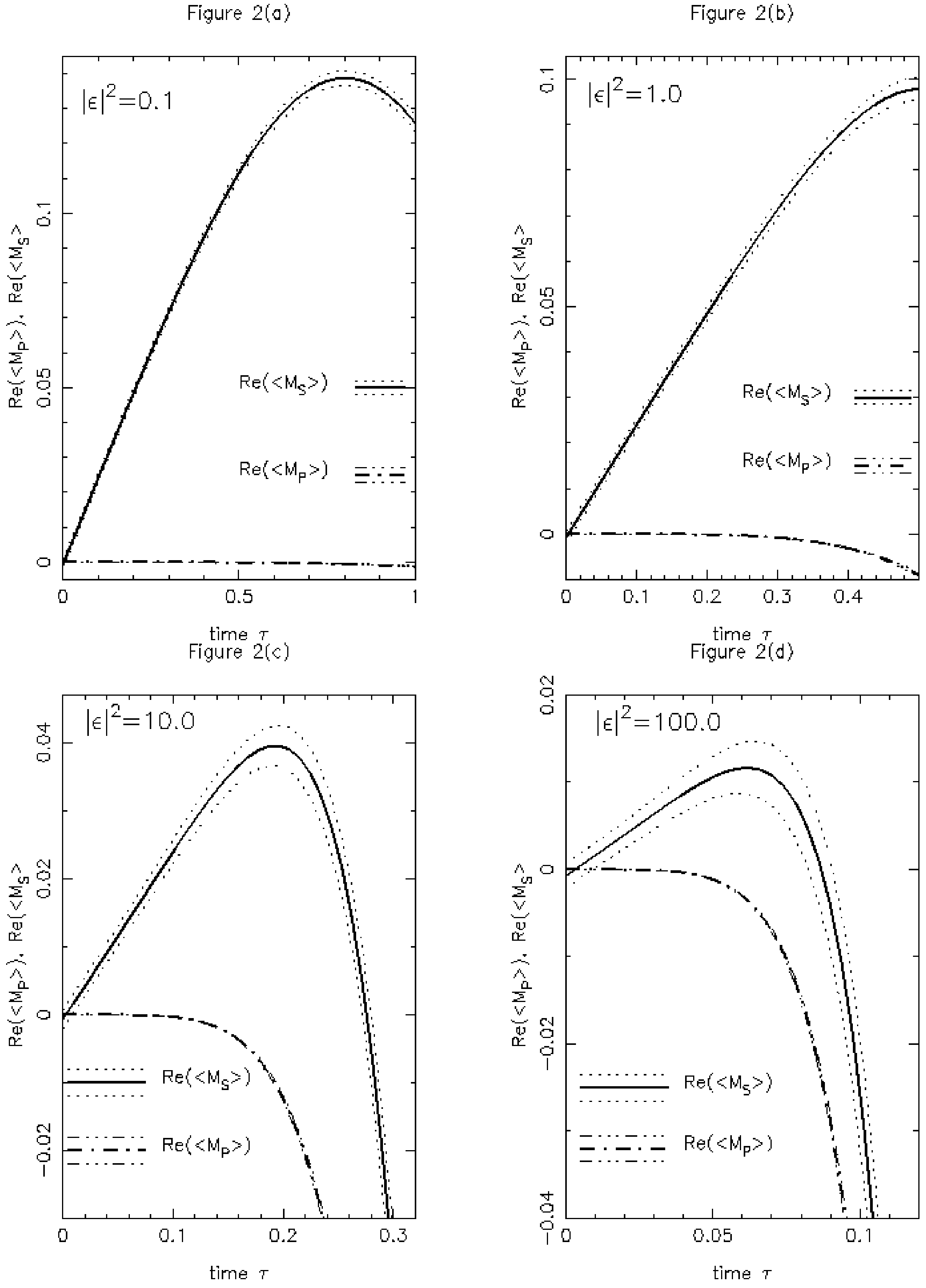


FIG. 2: **Figure 2:** Graphs showing the evolution of the third order moments $M_S, M_P$ as a function of time for the same initial conditions as on the previous figure, but with the scales chosen in order to highlight the differences in the initial evolution. The statistical errors due to the finite ensembles are indicated by the broken curves above and below the average values.