# NLS Solitons and the Few-Cycle Regime

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http://www.lsr.ph.ic.ac.uk/ http://www.kinsler.org/physics/ Talk; Abstract;

# Motivation

♦ Few cycle pulses are being generated in the laboratory – "*Five-optical-cycle pulse generation...*" Beddard et.al. (2000).

★ We can do pulse propagation using FDTD solution of Maxwell's equations; but envelope approaches are faster and more intuitive.

★ Brabec & Krausz (1997) derived an envelope SEWA eqn, extending the commonly used SVEA eqn into a few cycle regime. (see also Porras's (1999) slightly different SEWA eqn which better treats transverse effects.)

► *... but what happens to solitons in the few-cycle regime? Do they fall apart or do they stay robust?*

**Summary:** Solitons → Theory → Few-cycle Solitons → Colliding Solitons → Logical Solitons → Parametric Solitons → Spectral Overlap → Conclusions

# NLS Solitons

■ The Non-Linear Schrœdinger (NLS) equation governing soliton propagation is:

$$\partial_\xi A(\xi,\tau) = \imath\beta_1\partial_\tau A(\xi,\tau) - \frac{\beta_2}{2}\partial_\tau^2 A(\xi,\tau) + \frac{2\imath\pi}{n_0^2}A(\xi,\tau)^* A(\xi,\tau)^2, \quad (1)$$

$$A(\xi,\tau) = \eta \,\mathrm{sech}\left(\eta\left[\tau - v\xi\right]\right). \quad (2)$$

► Despite the apparently delicate balance between nonlinearity and dispersion, solitons are quite robust. If you try to propagate a vaguely soliton-like pulse, and generally it will shed all the non-soliton parts, leaving a "good" soliton.

► NB: Throughout this presentation, I'm going to assume a "perfect" medium – pure $\chi^{(3)}$ and only up to second order dispersion. Whilst unlikely to be realistic for true few-cycle pulses, I'm mainly interested in the extra FC effects, not an engineering simulation.

# Few-Cycle Theory

■ **3D wave eqn,** small transverse variation, plane polarized, dispersion by expanding $k$ about $\omega_0$, with $k(\omega)^2 = \tilde{\varepsilon}(\omega)\omega^2/c^2$; $z$ axis propagation, $\partial_\alpha \equiv \partial/\partial\alpha$:

$$\left(\partial_z^2 + \nabla_\perp^2\right) E(\vec{r},t) - \frac{\partial_t^2}{c^2}\int^t dt'\varepsilon_t(t-t')E(\vec{r},t') = \frac{4\pi}{c^2}\partial_t^2 P_{nl}(\vec{r},t)$$

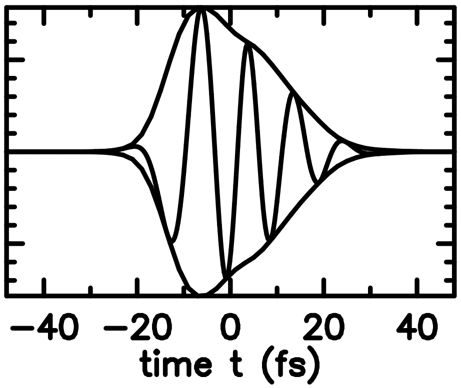
■ **Field ⇔ Envelope × Carrier**

Split the field $E$ (and polariz. $P_{nl}$) into an envelope $A$ (B) & carrier:

$$\begin{aligned} E(\vec{r},t) &= A(\vec{r}_\perp,z,t)e^{\imath(\beta_0 z - \omega_0 t + \psi_0)} + \text{c.c.} \\ P_{nl}(\vec{r},t) &= B(\vec{r}_\perp,z,t)e^{\imath(\beta_0 z - \omega_0 t + \psi_0)} + \text{c.c.}, \end{aligned}$$

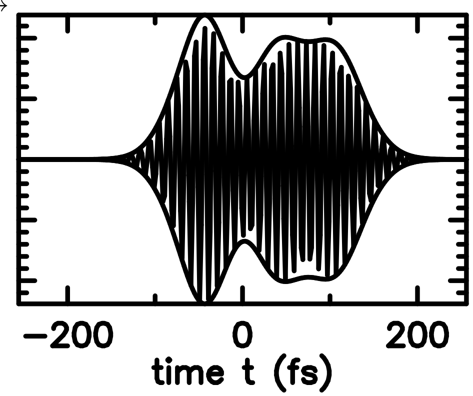
▶ Pulse envelopes tend to be less useful in the few cycle case, because they are less likely to be smooth (w.r.t. $\omega_0$).

▶ A given envelope represents many different field profiles, since there are many possible carrier phases.




←few-cycle

many-cycle →




## ■ Derive an envelope propagation equation...

□ Split the description into $A$ and $A^*$ pieces, and solve one.

□ Co-moving variables $\tau = \omega_0 (t - \beta_1 z)$, $\xi = \beta_0 z$;
define $\sigma = \omega_0\beta_1/\beta_0 = (\omega_0/\beta_0)/(1/\beta_1) = v_f/v_g$; and $n_0 = c\beta_0/\omega_0$.

□ Dispersion parameters $\gamma_n = \partial_\omega^n k(\omega)|_{\omega_\varepsilon} = \beta_n + \imath\alpha_n$.

$$(\beta_0/\omega_0)\,\hat{D}' \;=\; \left[ \imath\alpha_1 \left(\imath\partial_\tau\right) + \sum_{n=2}^{\infty} \frac{\gamma_n' \omega_0^{n-1}}{n!} \left(\imath\partial_\tau\right)^n \right].$$

## ■ An exact propagation equation for $A(\xi,\tau)$ with $B(\xi,\tau;A)$ ...

$$\partial_\xi A = \left(-\frac{\alpha_0}{\beta_0} + \imath \hat{D}'\right) A + \frac{\imath/\left(2\beta_0^2\right)}{\left(1+\imath\sigma\partial_\tau\right)} \nabla_\perp^2 A + \frac{2\imath\pi}{n_0^2} \frac{\left(1+\imath\partial_\tau\right)^2}{\left(1+\imath\sigma\partial_\tau\right)} B + \frac{T_{RHS}}{1+\imath\sigma\partial_\tau}.$$

▶ $T_{RHS} = 0 \Rightarrow$ generalised few-cycle approximation (GFEA) eqn.

▶ Backward propagating parts appear as a rapid modulation of the envelope; but are approx'ed away when enforcing "smoothness".

## ■ Approximations ...

$$T_{RHS} = \left[-\frac{\imath}{2}\partial_\xi^2 + \frac{\imath}{2}\left(\frac{\alpha_0}{\beta_0} - \imath\hat{D}'\right)^2\right] A(\xi,\tau) \approx 0.$$

**Slow Evolution:** $\partial_\xi^2 \ll 1$ **if** $\left|\partial_\xi \tilde{A}(\xi,\Omega)\right| \ll \left|\tilde{A}(\xi,\Omega)\right|,$

**Weak Dispersion:** $\partial_\tau \ll 1$ **if** $\left|\frac{\omega_0^m \gamma_m \Omega^m}{\beta_0 m!} \tilde{A}(\xi,\Omega)\right| \ll \left|\tilde{A}(\xi,\Omega)\right|,$

**Small Diffraction:** $\nabla_\perp^2$ **if** $\left(1+\sigma\Omega\right)\beta_0^2 w_0^2 \gg 1,$

**Weak Nonlinearity:** $B$ **if** $\frac{n_0^2}{2\pi}\frac{\left(1+\sigma\Omega\right)}{\left(1+\Omega\right)^2} \gg \frac{\left|\tilde{B}(\xi,\Omega;A)\right|}{\left|\tilde{A}(\xi,\Omega)\right|}.$

## ■ In order of increasing accuracy:

□ **SVEA:** *Slowly-Varying Envelope Approximation*

.
$$(1+\imath\partial_\tau)^2 / (1+\imath\sigma\partial_\tau) B \longrightarrow B \qquad \text{SVEA}$$

□ **SEWA:** *Slowly-Evolving Wave Approximation*

— see Brabec & Krausz, Porras.

$$(1+\imath\partial_\tau)\left[1+\frac{\imath(1-\sigma)\partial_\tau}{(1+\imath\sigma\partial_\tau)^2}\right] B \longrightarrow (1+\imath\partial_\tau) B \qquad \text{SEWA}$$

$$\frac{(1+\imath\partial_\tau)^2}{(1+\imath\sigma\partial_\tau)} B \longrightarrow (1+\imath[2-\sigma]+\partial_\tau) B \qquad \text{SEEA.}$$

□ **GFEA:** *Generalised Few-cycle Envelope Approx.*

— This is our new approximation; & includes all time-dependent corrections to the nonlinear polarization. Aka "Generalised Finite Envelope Approx" $\longrightarrow$ FPEA – Finite Pulse Envelope Approximation

## ■ Expansion of few-cycle polarization terms:

$$1+\imath(2-\sigma)\partial_\tau-(1-\sigma)^2\partial_\tau^2+\imath\sigma(1-\sigma)^2\partial_\tau^3+\mathcal{O}(\partial_\tau^4).$$

# Summary

## ■ I have ...

- ▶ ... derived exact envelope equations governing the propagation and nonlinear interaction of short pulses.
- ▶ ... done this without the SVEA, and without death by creeping approximation.
- ▶ ... clearly defined the form of the terms which are removed, and specified the necessary approximations.

## ■ This shows ...

- ▶ ... the "few cycle" effect is (mostly) a pulse profile dependent adjustment to the nonlinearity, which affects the phases and interactions of propagating fields.

## ■ I now ...

- ▶ ... apply the GFEA to NLS solitons, giving examples of how their propagation, collisions, and interactions change.
- ▶ ... apply the GFEA to parametric solitons, showing how their propagation alters.
- ▶ ... remark on situations in which different pulses have spectra that overlap.

# Few-Cycle Solitons

■ The leading order few-cycle correction to the nonlinear term in the NLS soliton propagation equation is:

$$\frac{\chi}{\omega_0}(2-\sigma)\,\partial_t\,|A|^2 A. \tag{3}$$

▶ Apart from the prefactor, this looks the same as the "self-steepening" perturbation studied by Biswas & Aceves, which results from the wavelength dependence of the nonlinearity.

▶ This type of term causes a velocity shift –

$$v = -\kappa+\Delta v = -\kappa-\varepsilon\eta^2\lambda, \tag{4}$$

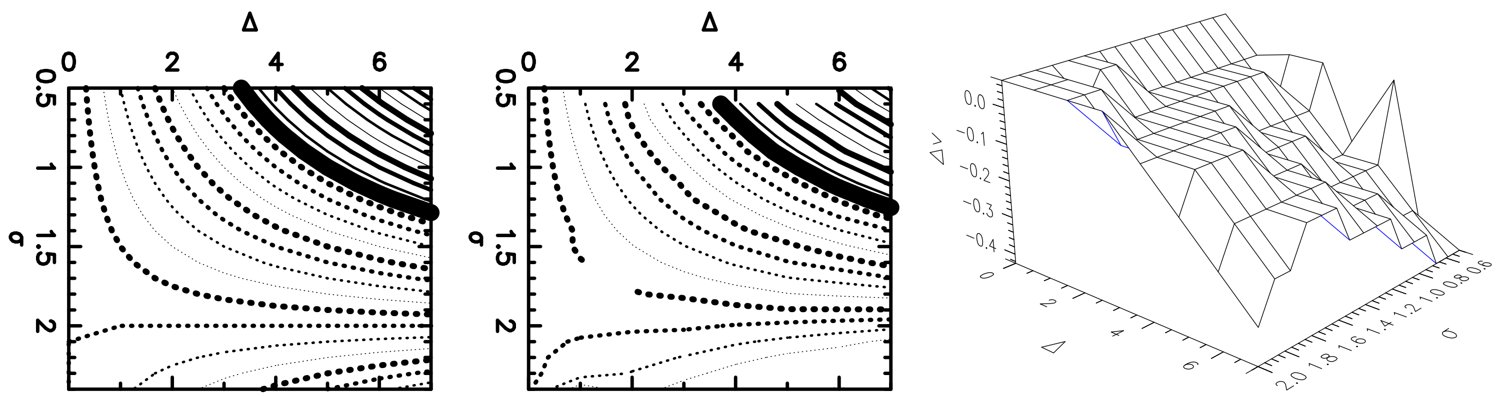
$$\text{Hence} \quad \Delta v = -\varepsilon\eta^2\lambda = -\eta^2\chi\,(2-\sigma)\,/\omega_0 \tag{5}$$

▶ Note that e.g. Biswas & Aceves and Wabnitz, Kodama & Aceves have done exhaustive studies of the effects of various perturbations on solitons and soliton-soliton collisions: including the "self steepening" term of the same form the first order few-cycle correction.

# *Faster! Faster!* Few-Cycle Solitons

## ■ How good is this first order velocity shift formula?

► Contour plots of the timing shifts as a function of group/phase velocity ratio $\sigma$ (down) versus pulse bandwidth multiplier $\Delta$ (across).
(a) First order Biswas & Aceves perturbation theory: $-\eta^2\chi\,(2-\sigma)\,/\omega_0$
(b) full few-cycle (GFEA) simulations.
► (c) Difference between 1st order and full Simulations: $\Delta v$.




(a) (b) (c)

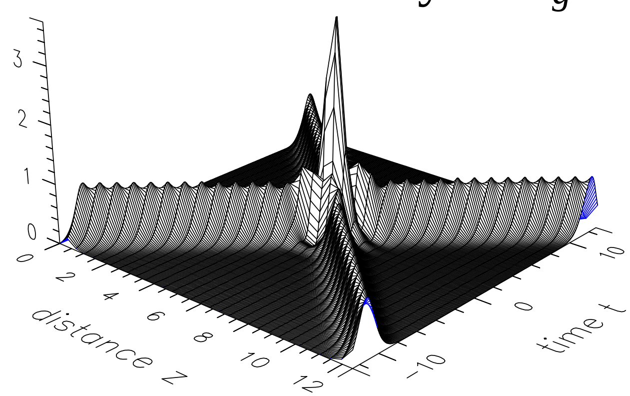
► Note a GFEA contour crosses the line $\sigma = 2.00$, whereas the B&A contour lies exactly along it – here the leading term $(2-\sigma)$ has vanished.

# *Colliding* Few-Cycle Solitons

■ Launch a pair of oppositely detuned solitons into a Kerr medium, so that the faster one will overtake the slower – then watch the outcome after the pulses have crossed over.

► For $v_f/v_g = \sigma = 1$, only the first order few-cycle term is non-zero, so we wouldn't expect to see much more than the velocity shift

► Note: the first order term isn't exactly a $\Delta v_g$ – it's $\partial_\tau A\,|A|^2$, not $\partial_\tau A$.

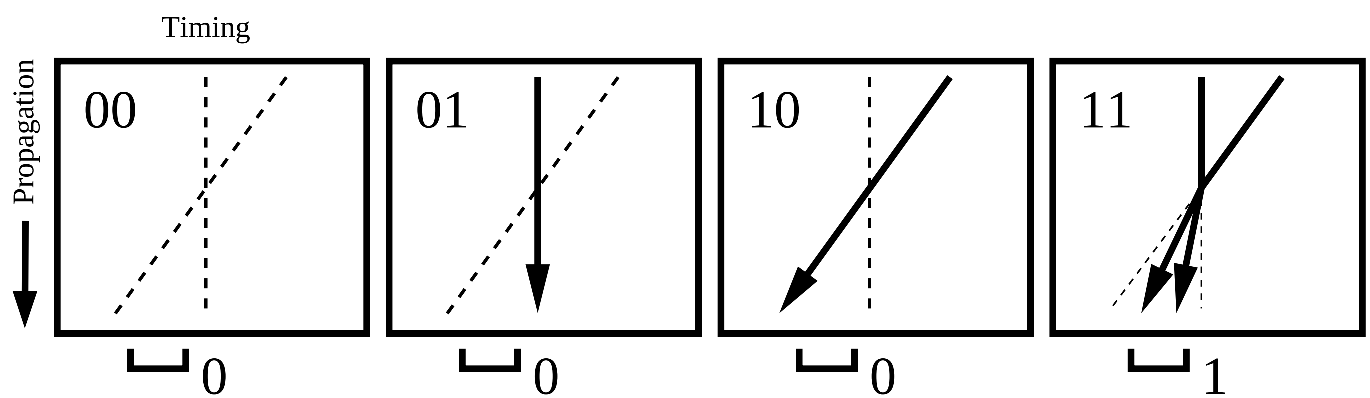



► Features caused by $\sigma \neq 1$ and higher order few-cycle effects certainly exist, but as yet I can't easily characterise them.

# *Logical* Few-Cycle Solitons

■ Coupled Solitons: We send two equal frequency, but differing polarization solitons down a birefringent fibre. Their differing propagation velocities mean that at some point the two cross over and interact. The interaction causes the solitons to undergo opposite frequency shifts.

▶ Logic: we can use one of the pulses as a control pulse. By applying a spectral filter that looks for a shifted pulse; we can see if both pulses were present, or if only one was (see expt. Ahn et. al. 1996).

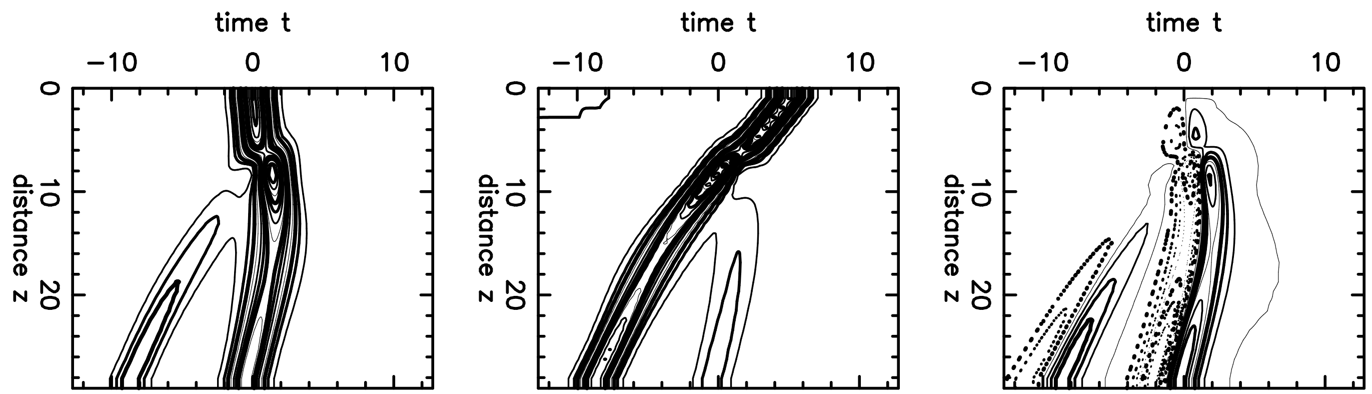



▶ NB: I've swapped the spectral filter with a timing one to make a simpler diagram. A change in soliton velocity (& so timing) is linked to a shift in frequency.

# *Logical* Few-Cycle Solitons (cont)

■ Here I show the "$1 \oplus 1 \rightarrow 1$" case, as the others are unremarkable. The simulaions are for 3 cycle solitons with $\sigma = 1$.

► *Left:* pulse A, ►► *Centre:* pulse B, ►►► *Right:* pulse A, GFEA−SVEA.




► The right hand difference graph shows how few-cycle effects alter the soliton propagation and interaction – solid contours are where the GFEA pulse is bigger than the SVEA, dashed contours the opposite.

► Few-Cycle: Of course few-cycle effects alter the interaction. But its character remains unaltered, and logic gate operation is largely unaffected – even at 2 cycles!.

# *Parametric* Few-Cycle Solitons

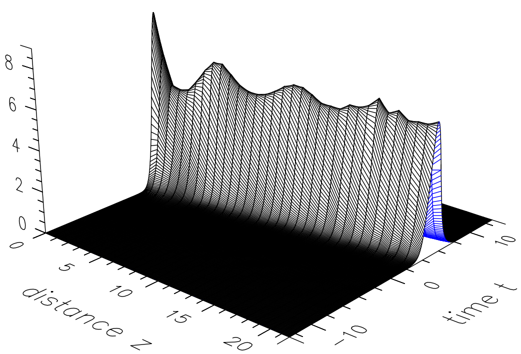
■ See e.g. Werner & Drummond 1993

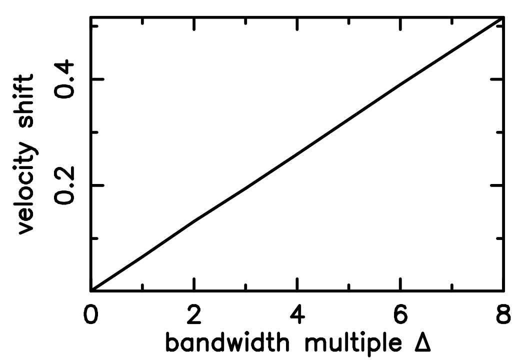
$$\partial_\xi \psi + \imath z_0 \left(\beta_{0\psi} - 2\beta_{0\phi}\right) + \frac{\imath}{2} \frac{\beta_{2\psi}}{\left|\beta_{2\phi}\right|} \partial_\tau^2 \psi = -\frac{1}{2}\phi^2, \tag{6}$$

$$\partial_\xi \phi + \frac{\imath}{2} \operatorname{sgn}(\beta_{2\phi}) \partial_\tau^2 \phi = \psi \phi^*, \tag{7}$$

$$\phi(z,t) = \phi_0 \operatorname{sech}^2(\kappa\tau) \exp(\imath \theta_\phi \xi); \quad \psi(z,t) = \psi_0 \operatorname{sech}^2(\kappa\tau) \exp(\imath \theta_\psi \xi). \tag{8}$$

◄ (L) 3 cycle ($\sigma = 1$) GFEA example; (R) velocity shifts ►



# ***Overlapping*** **Few-Cycle Solitons: (Short Pulses, Wide Spectra)**

■ Two solitons, at $\omega_1$ and $\omega_2$: the nonlinear polarization is

$$P = \chi E^3 = \chi \left(E_1 + E_2\right)^3 = \chi \left(A_1 e^{-\imath\omega_1 t} + A_2 e^{-\imath\omega_3 t} + \text{c.c.}\right)^3 \qquad (9)$$

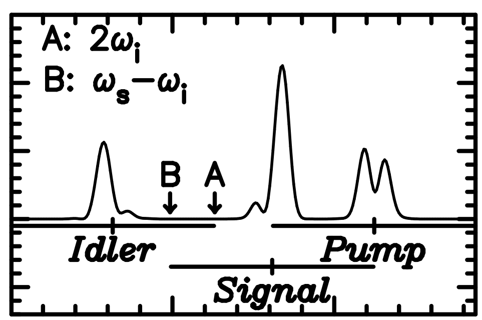
▶ We need to be aware of no-longer "off-resonant" polarization terms – and assign them to a suitable field component.

▶ Can we still maintain envelope smoothness with a single carrier component? (e.g. wide, double peaked spectra)

▶ Field envelope spectra can be so wide that they overlap.

▶ Pulse energy depends additionally on envelope gradient.

■ The optical parametric oscillator (OPO) nicely shows a typical multi-carrier few-cycle situation:

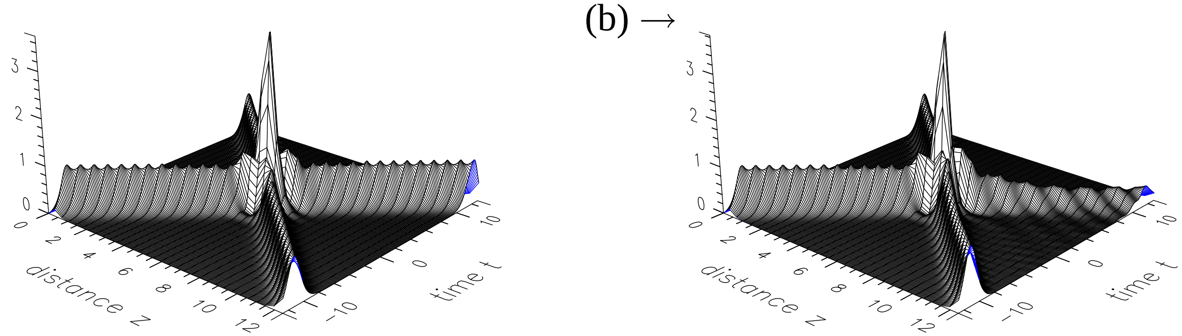


# ***Overlapping*** **II: (collisions & double envelopes)**

■ So: simulate a soliton collision with both in a single envelope, with opposite detunings; *and* simulate them with to coupled envelopes that have detuned carriers.

▶ Simulations: (a) single envelope (b) two envelope, recombined.

(a) → (b) →




■ The two simulations should give the same answer to within the numerical accuracy.

# Conclusions

▶ **Theory:** I derived a generalised few-cycle approximation (GFEA) for pulse propagation.

▶ **Theory:** few-cycle pulses get a "phase-twist" during propagation.

★ **NLS Solitons:** the first order few-cycle corrections dominate and induce a velocity shift

★ **NLS Solitons:** Even if colliding or interacting, the gross effects are still dominated by the "first-order" few-cycle velocity shift.

★ **NLS Logic:** Few-cycle effects do make a difference – but the logic gate still can operate.

★ **Parametric Solitons:** again a velocity shift, but the first order few-cycle corrections are secondary to higher order effects.

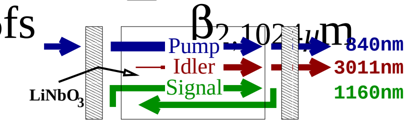
▶ **Next:** ...

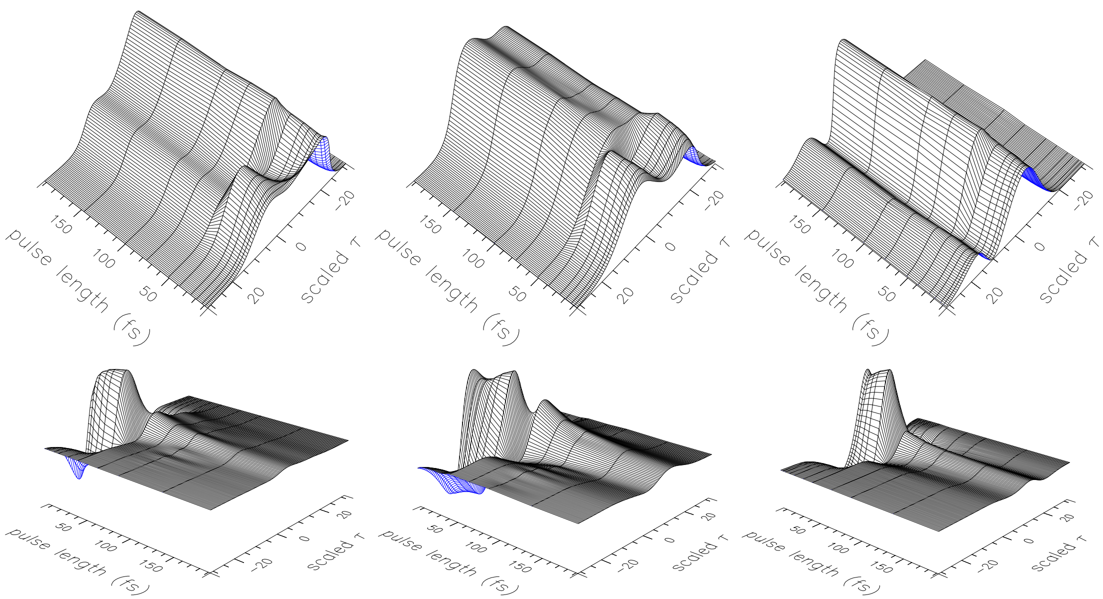
# Links


- **Kinsler & New:** Phys. Rev. A, in press, (2003)
  Preprint: http://arXiv.org/physics/0212016
  Detailed Calculation: http://arXiv.org/physics/0212014
  Web: http://www.qols.ph.ic.ac.uk/∼kinsle/
  Web: http://www.kinsler.org/physics/
- **Brabec & Krausz:** Phys. Rev. Lett. **78**, 3282 (1997)
  http://link.aps.org/abstract/PRL/v78/p3282
- **Porras:** Phys. Rev. A **60**, 5069 (1999)
  http://link.aps.org/abstract/PRA/v60/p5069
- **Beddard,** Ebrahimzadeh, Reid, Sibbett: Opt. Lett. **25**, 1052 (2000)
  http://ol.osa.org/abstract.cfm?id=62075
- **Biswas & Aceves:** J. Mod. Opt. **48**, 1135 (2001)
- **Wabnitz, Kodama & Aceves:** Opt. Fiber. Technol.**1**, 187 (1995)
- **Ahn et. al.:** *"Experimental demonstration of a low-latency fiber soliton logic gate"* J. Lightwave Techn. **14**, 1768 (1996).
- **Werner & Drummond:** J.O.S.A. B**10**, 2394 (1993).

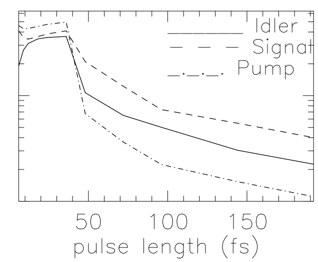
# Scaled Optical Parametric Oscillator

$$\frac{\text{Crystal Length}}{1024\mu\text{m}} = \frac{\text{Pulse Widths}}{48\text{fs}} = \frac{10\text{nJ}}{\text{Pump Energy}} = \frac{\text{Pump Delay}}{96\text{fs}} = \frac{\text{Dispersion}}{\beta_2\,1024\mu\text{m}}$$

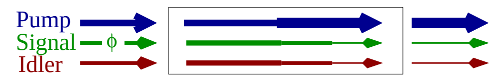


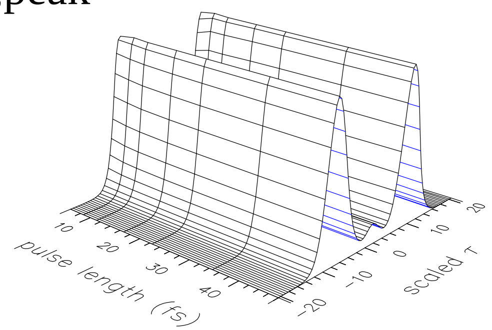

Idler Signal Pump

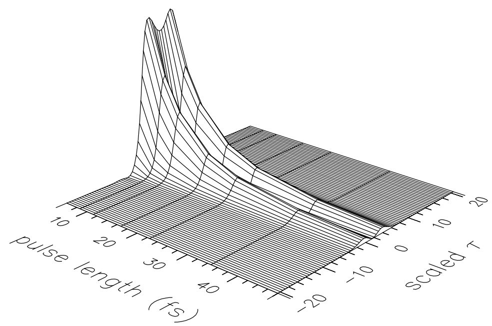


# Idealised OP De-amplification




$I_{\text{s,peak}}$: SVEA $\times 50$ GFEA $\times 1$




- Output signal intensities for $\phi_s = \pi/2$ and $\phi_p = \phi_i = 0$. Peak GFEA intensity at 48fs is a factor of $\sim 32$ larger than that of the SVEA model.

$\mathcal{E}_{\text{s,pulse}}$: SVEA GFEA